Semester 1 Cumulative   Name________________________________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Convert the angle in degrees to radians. Express answer as a multiple of π.

1) $-480°$ 1) ________

A) $-\frac{9\pi}{4}$ radians   B) $-\frac{7}{3}\pi$ radians   C) $-\frac{7\pi}{2}$ radians   D) $-\frac{8\pi}{3}$ radians

$180° = \pi$ radians. Let's use proportions to calculate the desired angle in radians.

$$\frac{-480}{180} = \frac{x}{\pi} \quad \Rightarrow \quad \frac{-480}{180}\pi = x \quad \Rightarrow \quad -\frac{8}{3}\pi = x$$

Answer D

Convert the angle in radians to degrees.

2) $\frac{11}{4}\pi$ 2) ________

A) $164°$ B) $65\pi°$ C) $495°$ D) $990°$

Use proportions to calculate the desired angle in degrees.

$$\frac{\frac{11}{4}\pi}{\pi} = \frac{x}{180} \quad \Rightarrow \quad \frac{11}{4} = \frac{x}{180} \quad \Rightarrow \quad \frac{11}{4} \cdot 180 = x \quad \Rightarrow \quad 495° = x$$

Answer C

Find a positive angle less than $360°$ or 2π that is coterminal with the given angle.

3) $\frac{17\pi}{5}$ 3) ________

A) $-\frac{17\pi}{5}$ B) $\frac{7\pi}{5}$ C) $\frac{12\pi}{5}$ D) $\frac{3\pi}{5}$

To get the desired coterminal angle, add or subtract 2π repeatedly until the value of θ is in the interval $(0, 2\pi)$.

$$\frac{17\pi}{5} - 2\pi = \frac{17\pi}{5} - \frac{10\pi}{5} = \frac{7\pi}{5} \text{ radians}$$

Answer B

Use the unit circle to find the value of the trigonometric function.

4) $\csc\frac{3\pi}{2}$ 4) ________

A) -1 B) 0 C) undefined   D) 1

$$\csc\frac{3\pi}{2} = \frac{1}{\sin\frac{3\pi}{2}} = \frac{1}{-1} = -1$$

Answer A

Use periodic properties of the trigonometric functions to find the exact value of the expression.

5) $\sin \frac{22\pi}{3}$ 5) ________

A) $\frac{\sqrt{3}}{2}$ B) -1 C) $-\frac{1}{2}$ D) $-\frac{\sqrt{3}}{2}$

$$\sin\left(\frac{22\pi}{3}\right) = \sin\left(\frac{22\pi}{3} - 6\pi\right) = \sin\left(\frac{22\pi}{3} - \frac{18\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Answer D

Further explanation: the reference angle for $\frac{4\pi}{3}$ in Q3 is: $\frac{4\pi}{3} - \pi = \frac{\pi}{3}$ and $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$. In Q3, the sine function is negative, so the desired cosine value is $-\frac{\sqrt{3}}{2}$.

Find a cofunction with the same value as the given expression.

6) $\cos 39°$ 6) ________

A) $\sin 51°$ B) $\csc 51°$ C) $\sin 39°$ D) $\sec 39°$

$\cos 39° = \sin(90° - 39°) = \sin 51°$ **Answer A**

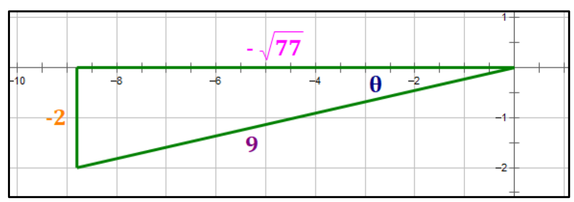
Number 7 is a long problem; it is on the next page. I know you want me to save paper!

Find the exact value of the indicated trigonometric function of θ.

8) $\sin \theta = -\frac{2}{9}$, $\tan \theta > 0$ Find $\sec \theta$. 8) ________

A) $-\frac{2\sqrt{77}}{77}$ B) $-\frac{\sqrt{77}}{9}$ C) $-\frac{9\sqrt{77}}{77}$ D) $\frac{\sqrt{9}}{2}$

Notice that $\sin\theta < 0$, $\tan\theta > 0$. Therefore θ is in $Q3$, where x and y are both negative.



$$\sin\theta = \frac{y}{r} = \frac{-2}{9}$$

Then, the horizontal leg must be:

$$x = -\sqrt{9^2 - (-2)^2} = -\sqrt{77}.$$ Then,

$$\sec\theta = \frac{1}{\cos\theta} = \frac{r}{x} = \frac{9}{-\sqrt{77}} = -\frac{9\sqrt{77}}{77}$$

Answer C

Solve the problem.

7) A surveyor is measuring the distance across a small lake. He has set up his transit on one side of the lake 150 feet from a piling that is directly across from a pier on the other side of the lake. From his transit, the angle between the piling and the pier is 50°. What is the distance between the piling and the pier to the nearest foot? 7) ________

A) 115 feet B) 179 feet C) 126 feet D) 96 feet

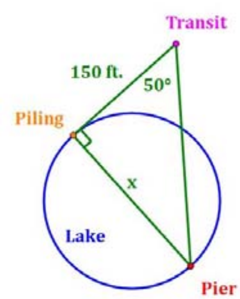
This problem is very difficult to understand. Let's see if we can make sense of it. Note that there are multiple interpretations of the problem and that they are all unsatisfactory.

- The problem does not say the lake is a circle, but if it is not, the problem cannot be solved. So, let's assume the lake is a circle.
- What is a transit? From www.surveyhistory.org, we get "The transit is used by the surveyor to measure both horizontal and vertical angles."

A transit →

A piling →

- What is a piling? From wisegeek.com, a piling is "a component in a foundation which is driven into the ground to ensure that the foundation is deep."
- We will need a right angle to solve a problem with only one length and angle given, so let's infer a 90˚ angle as shown in the following diagram.



Based on the diagram, which meets every criterion laid out in the problem, we now have:

$$\tan 50° = \frac{x}{150}$$

$$x = 150 \cdot \tan 50° = \mathbf{179\ ft.}$$ **Answer B**

Find the reference angle for the given angle.

9) $-\frac{11\pi}{12}$ 9) ________

A) $\frac{13\pi}{12}$ B) $\frac{11\pi}{12}$ C) $\frac{\pi}{12}$ D) $\frac{\pi}{24}$

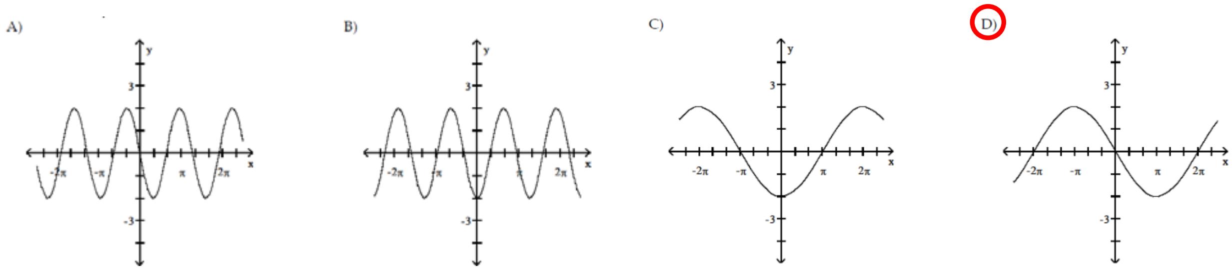
First, find the positive coterminal angle less than 2π radians.

$-\frac{11\pi}{12} + 2\pi = \frac{13\pi}{12}$. This angle is in Quadrant 3. So, the reference angle is:

$$\rho = \frac{13\pi}{12} - \pi = \frac{\pi}{12}$$ **Answer C**

Graph the following functions.

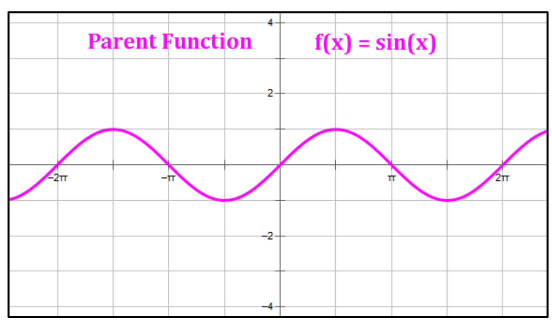
10) $y = -2 \sin \frac{1}{2} x$ **Standard Form:** $y = A\sin(Bx - C) + D$ 10) ______



Characteristic	$y = \sin x$	$y = -2\sin\frac{1}{2}x$
Amplitude $= \lvert A \rvert$	1	2 (negative)
Period $= \frac{2\pi}{B}$	2π	$2\pi \div \frac{1}{2} = 4\pi$
Phase Shift $= \frac{C}{B}$	0	0
Vertical Shift $= D$	0	0

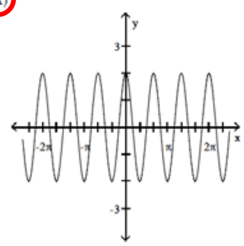
Characteristics of: $y = -2\sin\frac{1}{2}x$

- Period is 4π, so we are looking for a graph that has a compete wave over a distance of 4π. This narrows the choices to **C** and **D**.
- The amplitude is 2. That does not help.
- The graph is upside down, relative to the parent function because **A** is negative. That lead us to Answer **D**, which looks like an upside-down sine function.
- **Answer D**

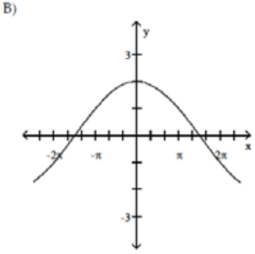


11) $y = 2\cos 3x$ **Standard Form:** $y = A\cos(Bx - C) + D$ 11) ______

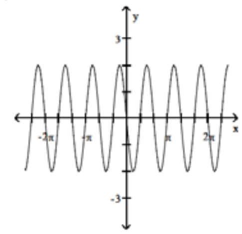
A)



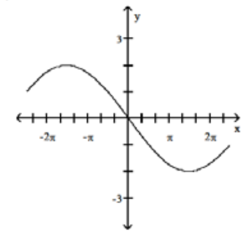
B)



C)



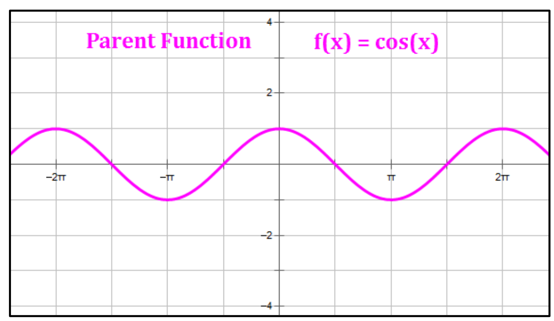
D)



Characteristic	$y = \cos x$	$y = 2\ \cos 3x$
Amplitude $= \lvert A \rvert$	1	2
Period $= \frac{2\pi}{B}$	2π	$2\pi \div 3 = \frac{2\pi}{3}$
Phase Shift $= \frac{C}{B}$	0	0
Vertical Shift $= D$	0	0

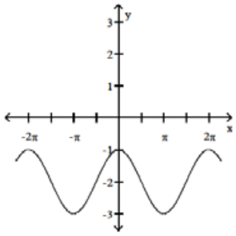
Characteristics of: $y = 2\cos 3x$

- Period is $\frac{2\pi}{3}$, so we are looking for a graph that has a compete wave over a distance of $\frac{2\pi}{3}$. This narrows the choices to **A** and **C**.
- The amplitude is 2. That does not help.
- The graph is a cosine function with no phase shift. That lead us to Answer **A**, whereas answer **C** has a phase shift.
- **Answer A**

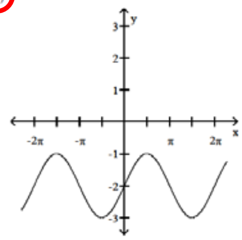


12) y = sin x – 2   **Standard Form:** $y = A\sin(Bx - C) + D$ 12) ________

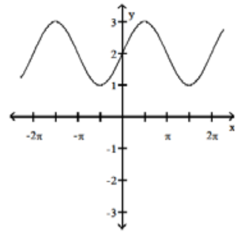
A)



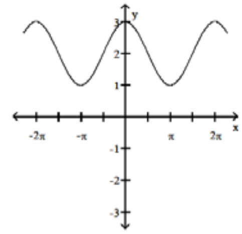
B)



C)



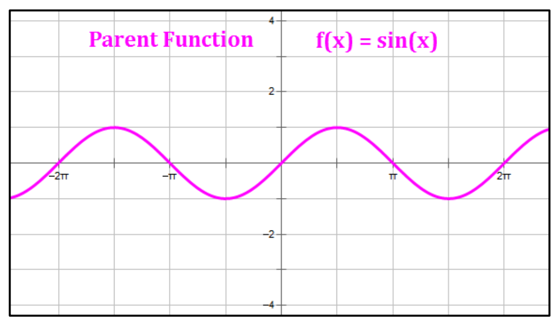
D)



Characteristic	$y = \sin x$	$y = \sin x - 2$
Amplitude $= \lvert A \rvert$	1	1
Period $= \frac{2\pi}{B}$	2π	2π
Phase Shift $= \frac{C}{B}$	0	0
Vertical Shift $= D$	0	-2

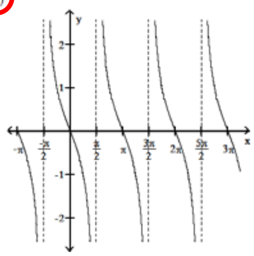
Characteristics of: $y = \sin x - 2$

- This function is the parent sine function, shifted down 2 units.
- **Answer B**

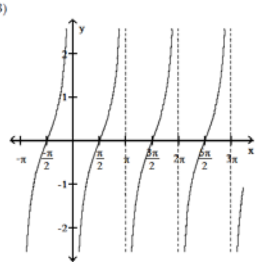


13) $y = -\tan x$ **Standard Form:** $y = A\tan(Bx - C) + D$ 13) ________

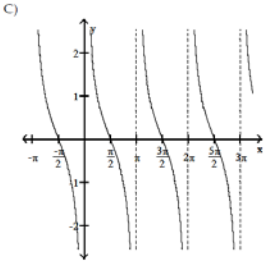
A)



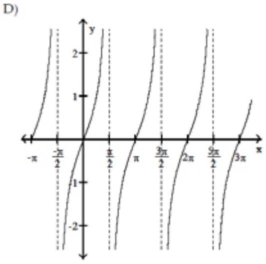
B)



C)



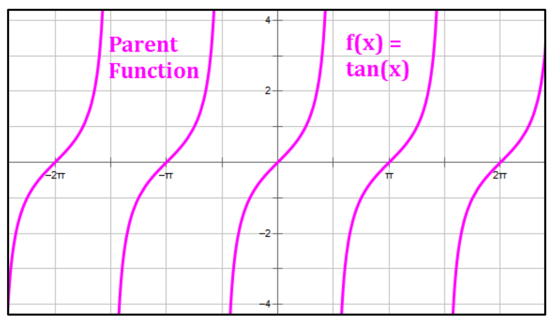
D)



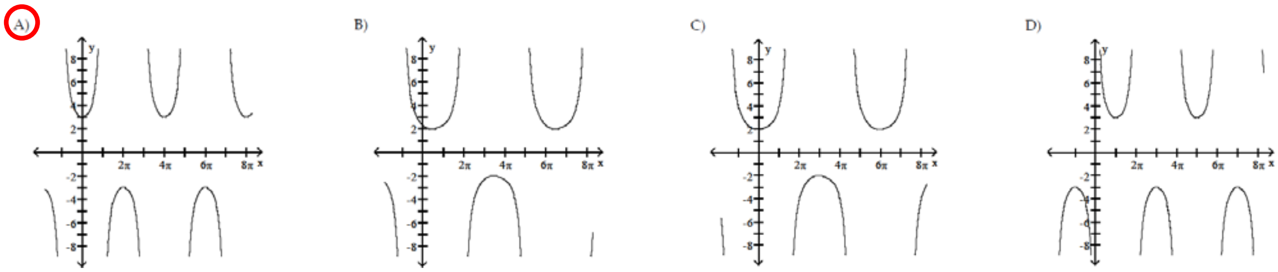
Characteristic	$y = \tan x$	$y = -\tan(x)$
Stretch $= \lvert A \rvert$	1	1 (negative)
Period $= \frac{\pi}{B}$	π	$\pi \div 1 = \pi$
Phase Shift $= \frac{C}{B}$	0	0
Vertical Shift $= D$	0	0

Characteristics of: $y = -\tan(x)$

- This function is simply the inverted tangent function.
- **Answer A**



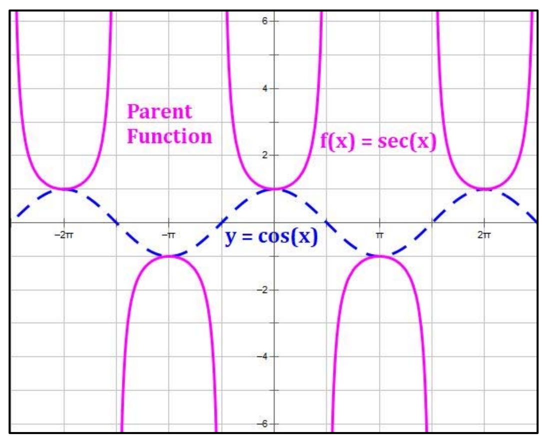
14) $y = 3\sec\frac{x}{2}$ **Standard Form:** $y = A\sec(Bx - C) + D$ 14) _______



Characteristic	$y = \sec x$	$y = 3\sec\left(\frac{1}{2}x\right)$
Stretch $= \lvert A\rvert$	1	3
Period $= \frac{2\pi}{B}$	2π	$2\pi \div \frac{1}{2} = 4\pi$
Phase Shift $= \frac{C}{B}$	0	0
Vertical Shift $= D$	0	0

Characteristics of: $y = 3\sec\left(\frac{x}{2}\right)$

- Period is 4π, so we are looking for a graph that has a compete wave (one regular U and one upside-down U) over a distance of 4π. This narrows the choices to **A** and **D**.
- The amplitude is 3. That does not help.
- The graph is a secant function with no phase shift. That lead us to Answer **A**, whereas answer **D** has a phase shift.
- **Answer A**



Find the exact value of the expression.

15) $\sin^{-1}\frac{\sqrt{3}}{2}$ 15) ______

A) $\frac{\pi}{4}$ B) $\frac{\pi}{3}$ C) $\frac{2\pi}{3}$ D) $\frac{3\pi}{4}$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is the angle that has a sine value equal to $\frac{\sqrt{3}}{2}$. $\sin^{-1} x$ is defined in Q1 and Q4.

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ **Answer B**

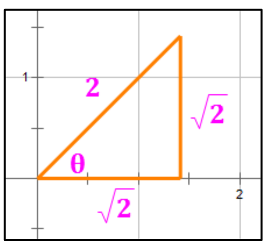
Use a sketch to find the exact value of the expression.

16) $\cot\left(\sin^{-1}\frac{\sqrt{2}}{2}\right)$ 16) ______

A) 2 B) $\frac{\sqrt{2}}{2}$ C) $\sqrt{2}$ D) 1

We are in Quadrant 1 because $\frac{\sqrt{2}}{2}$ is positive. Then, $\frac{y}{r} = \frac{\sqrt{2}}{2}$.

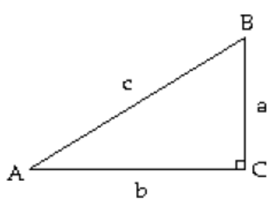
Calculate the horizontal leg of the triangle: $x = \sqrt{r^2 - y^2} = \sqrt{2^2 - \left(\sqrt{2}\right)^2} = \sqrt{2}$. Then draw.



Based on the diagram, then,

$$\cot\left(\sin^{-1}\left[\frac{\sqrt{2}}{2}\right]\right) = \cot\theta = \frac{x}{y} = \frac{\sqrt{2}}{\sqrt{2}} = \mathbf{1}$$ **Answer D**

Solve the right triangle shown in the figure. Round lengths to one decimal place and express angles to the nearest tenth of degree.



17) A = 59°, c = 48.4 17) ______

A) B = 31°, a = 24.9, b = 41.5
B) B = 59°, a = 41.5, b = 24.9
C) B = 31°, a = 41.5, b = 24.9
D) B = 59°, a = 24.9, b = 41.5

Since $m\angle A = 59°$, we know that $m\angle B = 90° - 59° = \mathbf{31°}$

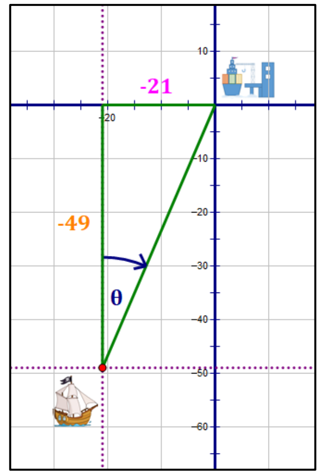
$$\sin A = \frac{a}{c} \quad \Rightarrow \quad \sin 59° = \frac{a}{48.4} \quad \Rightarrow \quad 48.4 \cdot \sin 59° = a = \mathbf{41.5}$$

There is no need to calculate length b. **Answer C**

Using a calculator, solve the following problems. Round your answers to the nearest tenth.

18) A ship is 21 miles west and 49 miles south of a harbor. What bearing should the captain set to sail directly to harbor? 18) _______

A) N 111.8° E B) N 66.8° E C) N 113.2 D) N 23.2° E



The diagram to the right illustrates this situation.

The ship wants to get to the port. We can see by the illustration that we need to calculate the angle θ, and that this angle will give us the bearing in the **NE** direction that the ship needs to travel.

The bearing will be read as an angle that begins North and moves to the East.

$$\tan\theta = \frac{x}{y}$$

$$\theta = \tan^{-1}\left(\frac{x}{y}\right) = \tan^{-1}\left(\frac{-21}{-49}\right) = 23.2°$$

Then, **the bearing that the ship must follow is: $\mathbf{N\ 23.2°\ E}$**

Answer D (not Answer A as shown in the answer key)

An object is attached to a coiled spring. The object is pulled down (negative direction from the rest position) and then released. Write an equation for the distance of the object from its rest position after t seconds.

19) amplitude = 11 cm; period = 3π seconds 19) _______

A) $d = -11\cos\frac{2}{3}\pi t$ B) $d = -11\sin\frac{2}{3}\pi t$ C) $d = -11\cos\frac{2}{3}t$ D) $d = -3\cos\frac{2}{11}t$

The spring will start at a y-value of -11 (since it is pulled down with an amplitude of 11), and oscillate between -11 and $+11$ (absent any other force) over time. A good representation of this would be a cosine curve with lead coefficient $a = -11$.

The period of the function is 3π seconds. So, we get:

$$f = \frac{1}{\text{period}} = \frac{1}{3\pi} \quad \text{and} \quad \omega = 2\pi f = 2\pi \cdot \frac{1}{3\pi} = \frac{2}{3}$$

The resulting equation, then, is: $\boldsymbol{d = -11\cos\left(\frac{2}{3}t\right)}$ **Answer C**

Note: ω is the lower-case Greek letter omega. The upper-case omega is Ω.

Find the unit vector that has the same direction as the vector v.

20) $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$ 20) _______

A) $\mathbf{u} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}$ B) $\mathbf{u} = \frac{5}{4}\mathbf{i} + \frac{5}{3}\mathbf{j}$ C) $\mathbf{u} = -\frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}$ D) $\mathbf{u} = 20\mathbf{i} + 15\mathbf{j}$

The unit vector is: $\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{4\mathbf{i}+3\mathbf{j}}{\sqrt{4^2+3^2}} = \frac{4\mathbf{i}+3\mathbf{j}}{5} = \frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}$ **Answer A**

Write the vector v in terms of i and j whose magnitude $\|\mathbf{v}\|$ and direction angle θ are given.

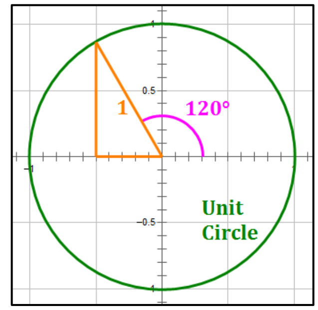
21) $\|\mathbf{v}\| = 10$, $\theta = 120°$ 21) ______

A) $\mathbf{v} = -5\mathbf{i} + 5\sqrt{3}\mathbf{j}$ B) $\mathbf{v} = 5\mathbf{i} - 5\sqrt{3}\mathbf{j}$

C) $\mathbf{v} = -5\sqrt{2}\mathbf{i} + 5\sqrt{2}\mathbf{j}$ D) $\mathbf{v} = 5\sqrt{3}\mathbf{i} - 5\mathbf{j}$

The unit vector in the direction $\theta = 120°$ is:

$$\langle \cos 120°, \sin 120° \rangle = \langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \rangle = -\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$$



Multiply this by $\|\mathbf{v}\| = \mathbf{10}$ to get $\mathbf{v}$:

$$\mathbf{v} = 10\left(-\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right) = -5\mathbf{i} + 5\sqrt{3}\,\mathbf{j}$$

Answer A

Find the specified vector or scalar.

22) $\mathbf{u} = -9\mathbf{i} - 4\mathbf{j}$, $\mathbf{v} = -4\mathbf{i} + 8\mathbf{j}$; Find $\mathbf{u} + \mathbf{v}$. 22) ______

A) $-14\mathbf{i} + 4\mathbf{j}$ B) $5\mathbf{i} + 4\mathbf{j}$ C) $-13\mathbf{i} + 4\mathbf{j}$ D) $-5\mathbf{i} - 14\mathbf{j}$

To add vectors, simply line them up vertically and add:

$$\mathbf{u} = \langle -9, -4 \rangle$$
$$\mathbf{v} = \langle -4, \ \ 8 \rangle$$

$$\mathbf{u} + \mathbf{v} = \langle -9 - 4, -4 + 8 \rangle$$

$$\mathbf{u} + \mathbf{v} = \langle -13, 4 \rangle = -13\mathbf{i} + 4\mathbf{j}$$ **Answer C**

The rectangular coordinates of a point are given. Find polar coordinates of the point. Express θ in radians.

23) $(6\sqrt{3}, 6)$ 23) ______

A) $\left(6, \frac{\pi}{6}\right)$ B) $\left(6, \frac{\pi}{3}\right)$ C) $\left(12, \frac{\pi}{3}\right)$ D) $\left(12, \frac{\pi}{6}\right)$

This point lies in quadrant 1, so,

$$r = \sqrt{x^2 + y^2} = \sqrt{(6\sqrt{3})^2 + 6^2} = \sqrt{144} = 12$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{6}{6\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

So, the polar coordinates are: $\left(12, \frac{\pi}{6}\right)$ **Answer D**

Important: the angle calculated using the $\tan^{-1} x$ function will always be in Q1 or Q4. If the rectangular point lies in Q2 or Q3, you will need to add π radians to $\tan^{-1} x$ to obtain the correct polar coordinates.

Convert the polar equation to a rectangular equation.

24) $r = -4 \cos \theta$ 24) _______

A) $x = -4$
B) $(x + 2)^2 + y^2 = 4$
C) $(x - 2)^2 + y^2 = 16$
D) $x^2 + y^2 = 4$

Substitute $\cos\theta = \frac{x}{r}$ and $r^2 = x^2 + y^2$

$r = -4\left(\frac{x}{r}\right)$

$r^2 = -4x$

$x^2 + y^2 = -4x$

$x^2 + 4x \quad + y^2 = 0$ Next, complete the square.

$(x^2 + 4x + 4) + y^2 = 4$

$(x+2)^2 + y^2 = 4$ **Answer B**

Find the x-intercepts of the polynomial function. State whether the graph crosses the x-axis, or touches the x-axis and turns around, at each intercept.

25) $f(x) = (x + 1)(x - 6)(x - 1)^2$ 25) _______

A) 1, crosses the x-axis;
−6, touches the x-axis and turns around;
−1, touches the x-axis and turns around

B) −1, crosses the x-axis;
6, crosses the x-axis;
1, touches the x-axis and turns around

C) 1, crosses the x-axis;
−6, crosses the x-axis;
−1, touches the x-axis and turns around

D) −1, crosses the x-axis;
6, crosses the x-axis;
1, crosses the x-axis

A polynomial function will pass through the x-axis if its multiplicity at that root is odd.
A polynomial function will touch the x-axis and turn around if its multiplicity at that root is even.

$(x+1) \Rightarrow$ Root is: $-1 \Rightarrow$ Multiplicity is: $1 \Rightarrow$ Graph passes through x-axis

$(x-6) \Rightarrow$ Root is: $6 \Rightarrow$ Multiplicity is: $1 \Rightarrow$ Graph passes through x-axis

$(x-1)^2 \Rightarrow$ Root is: $1 \Rightarrow$ Multiplicity is: $2 \Rightarrow$ Graph touches and turns around

Answer B

Solve the problem.

26) A deposit of $5000 is made in an account that earns 9% interest compounded quarterly. The balance in the account after n quarters is given by the sequence 26) _______

$$a_n = 5000\left(1 + \frac{0.09}{4}\right)^n \qquad n = 1, 2, 3, \ldots$$

Find the balance in the account after 24 quarters.

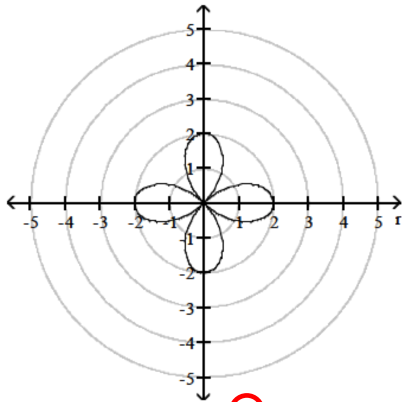
A) $8389.83
B) $8528.83
C) $8607.83
D) $8588.83

In this case, $n = 24$. Notice that they changed the formula on us. However, since the interest rate is quarterly $\left(\frac{.09}{4}\right)$, the number of periods should be the number of quarters. Then,

$$A = 5000\left(1 + \frac{.09}{4}\right)^{24} = \$8{,}528.83$$ **Answer B**

The graph of a polar equation is given. Select the polar equation for the graph.

27) 27) ______



A) $r = 2\sin(2\theta)$ B) $r = 2\cos(2\theta)$ C) $r = 2 + \cos(2\theta)$ D) $r = 2$

How should one approach this problem without an intimate knowledge of polar graphs?

Look at the possible answers:

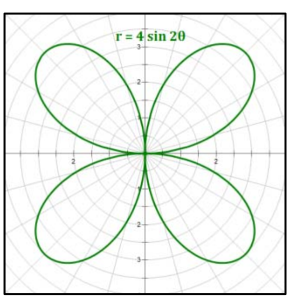
- Answer **C** requires the graph to flow between r-values of 1 and 3 because cosine oscillates between 1 and -1. It is out.
- Answer **D** is a circle of radius 2. It is out.
- That leaves Answers **A** and **B**, which will have different graphs. Try a point that fits the equation on one of the graphs and see if it fits.
- To tell the difference between the graphs in Answers **A** and **B**, we should try the angle $\theta = 45°$ because the two equations will have different r-values at that angle. We know this by looking at the two sample rose curves shown below (taken from the Trigonometry Handbook, v 2.2, page 74).

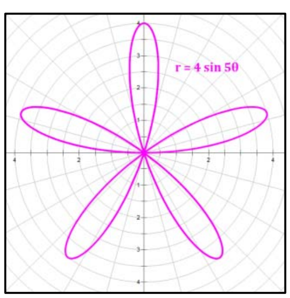
At $\theta = 45°$, find the value of r.

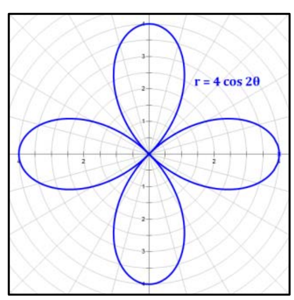
Answer **A**: $r = 2\sin 2\theta = 2\sin(2\cdot 45)° = 2\sin 90° = 2\cdot 1 = 2$. This is NOT on the graph.

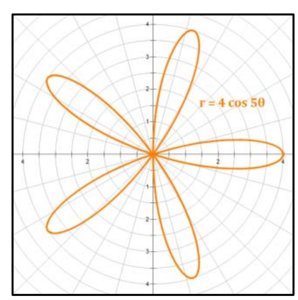
Answer **B**: $r = 2\cos 2\theta = 2\cos(2\cdot 45)° = 2\cos 90° = 2\cdot 0 = 0$. This IS on the graph.
Therefore, this this is the graph of $\boldsymbol{r = 2\cos 2\theta}$ **Answer B**

The four Rose Forms:





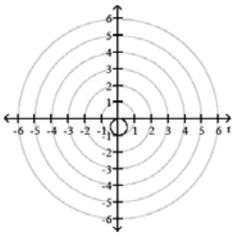




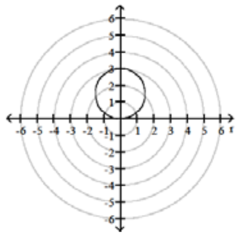
Graph the polar equation.

28) $r = 3 + \sin \theta$ 28) _______

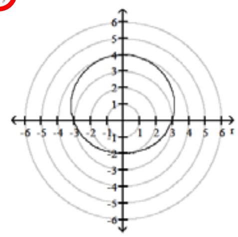
A)



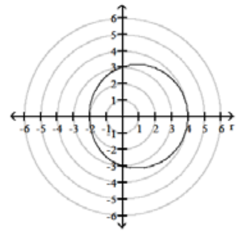
B)



C)



D)



How should one approach this problem without an intimate knowledge of polar graphs?

Look at the equation:

- $r = 3 + \sin \theta$ requires the graph to flow between r-values of 2 and 4 because sine oscillates between 1 and -1. So, Answers **A** and **B** are out.
- That leaves Answers **C** and **D**, which will have different graphs. Try a point on one of the graphs and see if it fits. To tell the difference between the graphs we should try the angle $\theta = 180°$ because the two graphs shown have different r-values at that angle.

$r = 3 + \sin \theta = 3 + \sin 180° = 3 + 0 = 3$. Note that the point $(3, 180°)$ is on Graph **C** but is not on Graph **D**.

Therefore, $r = 3 + \sin \theta$ is the equation of Graph **C.** **Answer C**

Determine whether the equation defines y as a function of x.

29) $y = -\sqrt{x - 5}$ 29) _______

A) y is a function of x B) y is not a function of x

An equation defines a function if it provides exactly one y-value for each x-value in its domain.

This equation does that. Also, notice that there is no y^2 term and there is no " $\pm$ " in the equation. So, this equation is a function. **Answer A**

Evaluate the function at the given value of the independent variable and simplify.

30) $g(x) = 4x + 2$; $g(x + 1)$ 30) _______

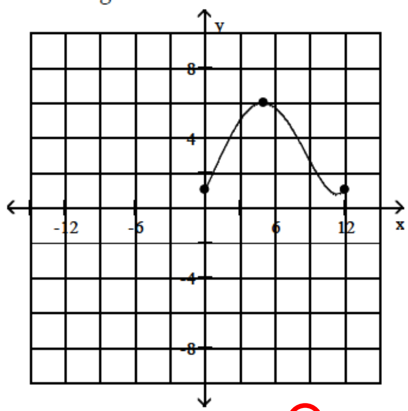
A) $4x + 2$ B) $4x + 6$ C) $4x - 1$ D) $\frac{1}{4}x + 2$

$g(x) = 4x + 2$

$g(x + 1) = 4(x + 1) + 2 = 4x + 4 + 2 = \mathbf{4x + 6}$ **Answer B**

Identify the intervals where the function is changing as requested.

31) Increasing 31) ______



A) (1, 6) B) (0, 5) C) (0, 6) D) (1, 5)

A function is increasing at a point where y-values immediately to the left of the point are lower and y-values immediately to the right of the point are higher. The areas immediately to the left and right of a point on a graph are called a **neighborhood** of the point.

So, the function shown on the graph appears to be increasing on the interval from 0 to 5 and also on the interval from 11 to 12. The answer that makes the most sense is Answer **B**, even though it ignores the second interval where the function increases.

Answer B

Evaluate the piecewise function at the given value of the independent variable.

32) $h(x) = \begin{cases} \dfrac{x^2+1}{x-8} & \text{if } x \neq 8 \\ x-4 & \text{if } x = 8 \end{cases}$; h(8) 32) ______

A) 12 B) 4 C) undefined D) −4

To find the value of $h(8)$, we must look at the conditions given in the definition of the piecewise function, i.e., the "if" parts of the definition. In this problem, since $x = 8$, we must use the bottom of the two lines of given. So,

$h(x) = x - 4 \quad \Rightarrow \quad f(8) = 8 - 4 = \mathbf{4}$ **Answer B**

Solve the problem.

33) Suppose a car rental company charges $146 for the first day and $96 for each additional or partial day. Let S(x) represent the cost of renting a car for x days. Find the value of S(5.5). 33) ______

A) $578 B) $528 C) $626 D) $674

Charge for day 1:	$146	
Charge for days 2-5:	384	($96 per day for four days)
Charge for part of day 6:	96	
Total:	**$626**	**Answer C**

Find and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$ for the given function.

34) $f(x) = x^2 + 2x - 9$ 34) ______

A) $2x + h + 2$
B) $2x + h - 9$
C) 1
D) $\frac{2x^2 + 2x + 2xh + h^2 + h - 18}{h}$

Note that this expression gives the slope of the secant line connecting two points whose x-values are h units apart. Is forms part of the definition of the derivative in Calculus.

$$\frac{f(x+h)-f(x)}{h} = \frac{[(x+h)^2 + 2(x+h) - 9] - [x^2 + 2x - 9]}{h}$$

$$= \frac{(x^2 + 2hx + h^2) + (2x + 2h) - 9 - [x^2 + 2x - 9]}{h}$$

$$= \frac{x^2 + 2hx + h^2 + 2x + 2h - 9 - x^2 - 2x + 9}{h}$$

$$\frac{x^2 - x^2 + 2hx + h^2 + 2x - 2x + 2h - 9 + 9}{h}$$

$$= \frac{2hx + h^2 + 2h}{h}$$

$$= \mathbf{2x + h + 2}$$ **Answer A**

Note: If you know a little Calculus, you know that $\frac{d}{dx}(x^2 + 2x - 9) = 2x + 2$, which must be a part of the solution, and that there would be no denominator in the solution. This points directly to Answer **A**, without doing all the complicated Algebra.

Find functions f and g so that $h(x) = (f \cdot g)(x)$.

35) $h(x) = \frac{8}{\sqrt{8x + 10}}$ 35) ______

A) $f(x) = \sqrt{8x + 10}$, $g(x) = 8$
B) $f(x) = 8/x$, $g(x) = 8x + 10$
C) $f(x) = 8$, $g(x) = \sqrt{8 + 10}$
D) $f(x) = 8/\sqrt{x}$, $g(x) = 8x + 10$

One of the reasons students find these problems difficult is that there are an infinite number of ways to express f and g. We will stick to the most obvious solutions. Also, note that the dot between the f and the g appears to be a multiplication sign, $\cdot$. It is not. It is the function composition sign, $\circ$.

$h(x)$ appears to have the function $y = 8x + 10$ inside the function $y = \frac{8}{\sqrt{x}}$. We let the inside function be the function closest to x in the expression $(f \circ g)(x)$, that is, $g(x)$. We let the other function be $f(x)$. So,

$$\boldsymbol{f(x) = \frac{8}{\sqrt{x}}} \quad \text{and} \quad \boldsymbol{g(x) = 8x + 10}$$ **Answer D**

Find the inverse of the one-to-one function.

36) $f(x) = \sqrt{x + 5}$ 36) ______

A) $f^{-1}(x) = \frac{1}{x^2 - 5}$ B) $f^{-1}(x) = x - 5$ C) $f^{-1}(x) = x^2 - 5$ D) $f^{-1}(x) = x^2 + 5$

To find an inverse function, switch the x and y in the original function and solve for y.

$$x = \sqrt{y+5} \quad \Rightarrow \quad x^2 = y + 5 \quad \Rightarrow \quad x^2 - 5 = y$$

So,

$\boldsymbol{f^{-1}(x) = x^2 - 5}$ **Answer C**

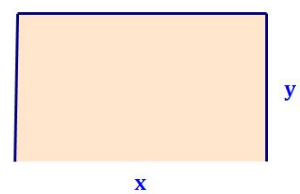
Solve the problem.

37) You have 104 feet of fencing to enclose a rectangular plot that borders on a river. If you do not fence the side along the river, find the length and width of the plot that will maximize the area. 37) ______

A) length: 52 feet, width: 26 feet B) length: 78 feet, width: 26 feet
C) length: 26 feet, width: 26 feet D) length: 52 feet, width: 52 feet

Let the rectangular plot have dimensions x by y. Then the perimeter is $P = x + 2y$ because one side of the plot does not need fencing (due to the river). The area is $A = xy$.

We are given that $P = 104$, and we are asked to maximize A.



Then,

$$104 = x + 2y \quad \Rightarrow \quad 104 - x = 2y \quad \Rightarrow \quad y = 52 - \frac{x}{2}$$

Substitute this into the formula for the area:

$A = xy = x\left(52 - \frac{x}{2}\right) = -\frac{1}{2}x^2 + 52x$ which is a parabola (upside down). Its maximum value is at the vertex. At the vertex, we have:

$$x = \frac{-b}{2a} = \frac{-52}{2\left(-\frac{1}{2}\right)} = 52 \text{ feet} \quad \Rightarrow \quad y = 52 - \frac{x}{2} = 52 - 26 = 26 \text{ feet}$$

Therefore, the dimensions that maximize the area of the box are **52 feet by 26 feet**, with the one side parallel to the river being the 52 feet. **Answer A**

Trick for these rectangular plot problems. No matter how many fences there are, the total length of the vertical (in the drawing) fences will equal half of the total fencing. The total length of the horizontal (in the drawing) fences will also equal half of the total fencing.

So, in this problem, we have $104 \div 2 = 52$ feet of horizontal fence, and 52 feet of vertical fence. Since there is only one horizontal fence, it will be 52 feet long. Since there are two vertical fences, they will each be $52 \div 2 = 26$ feet long.

Lesson: this is a multiple-choice problem, so there is no need to do the Algebra. Just use the trick mentioned here.

Find the zeros for the polynomial function and give the multiplicity for each zero. State whether the graph crosses the x–axis or touches the x–axis and turns around, at each zero.

38) $f(x) = x^3 + x^2 - 12x$ 38) ______

A) – 4, multiplicity 2, touches the x–axis and turns around
3, multiplicity 1, crosses the x–axis

B) 0, multiplicity 1, crosses the x–axis
4, multiplicity 1, crosses the x–axis
–3, multiplicity 1, crosses the x–axis

C) 0, multiplicity 1, crosses the x–axis
– 4, multiplicity 1, crosses the x–axis
3, multiplicity 1, crosses the x–axis

D) 0, multiplicity 1, touches the x–axis and turns around;
– 4, multiplicity 1, touches the x–axis and turns around;
3, multiplicity 1, touches the x–axis and turns around

We need to factor $f(x)$ in order to determine the roots (zeros) and their multiplicities.

$$x^3 + x^2 - 12x = x(x^2 + x - 12) = x(x+4)(x-3)$$

So, the **roots** of $f(x)$ are $x = \{\mathbf{0}, \mathbf{-4}, \mathbf{3}\}$.

Multiplicity is the exponent on the term that generates the root. All three terms of the factored form of $f(x)$ have powers of 1. Therefore, **all three roots have multiplicities of 1**.

Multiplicities that are odd result in the curve **passing through the x-axis** at the corresponding root. Multiplicities that are even result in the curve **touching the x-axis and turning around** at the corresponding root.

Therefore, **the curve passes through the x-axis at all three roots.**

Answer C

Divide using synthetic division.

39) $\dfrac{2x^3 - 13x^2 + 12x + 15}{x - 5}$ 39) ______

A) $\frac{2}{5}x^2 - \frac{13}{5}x + \frac{12}{5}$

B) $-2x^2 + 5x - 3$

C) $2x^2 - 3x - 3$

D) $-2x^2 - 5x + 3$

$$\begin{array}{r|rrrr} 5 & 2 & -13 & 12 & 15 \\ & & 10 & -15 & -15 \\ \hline & 2 & -3 & -3 & \boxed{0} \end{array}$$

The result of the division begins with one less degree than the original dividend, and uses the coefficients obtained via synthetic division. That is:

$\mathbf{2x^2 - 3x - 3}$ **Answer C**

Solve the polynomial equation. In order to obtain the first root, use synthetic division to test the possible rational roots.

40) $x^3 + 6x^2 - x - 6 = 0$ 40) ______

A) {-1, -2, -3} B) {1, -1, 6} C) {1, -2, 3} D) {1, -1, -6}

This is a multiple-choice question, so we can ignore how it says to do the work if we want to. However, it is good to practice synthetic division, so let's do that.

Possible rational roots are of the form $\frac{p}{q}$ where p is a factor of the constant term and q is a factor of the lead coefficient. So, possible rational roots are: $\pm\frac{1,2,3,6}{1} = \pm1, \pm2, \pm3, \pm6$. Let's consider them:

- $+1$ will work because the sum of the coefficients is zero.
- -1 will work because the sum of the coefficients of the odd degree terms $(1 - 1 = 0)$ is equal to the sum of the coefficients of the even degree terms $(6 - 6 = 0)$.

Let's use synthetic division with 1 as a root (or "zero").

$$\begin{array}{r|rrrr} 1 & 1 & 6 & -1 & -6 \\ & & 1 & 7 & 6 \\ \hline & 1 & 7 & 6 & \boxed{0} \end{array}$$

The remaining polynomial is $x^2 + 7x + 6 = (x + 1)(x + 6) \quad \Rightarrow \quad x = \{-1, -6\}$

So, the **complete solution** of the polynomial is the set of zeros: $\boldsymbol{x = \{-6, -1, 1\}}$ **Answer D**

Find a rational zero of the polynomial function and use it to find all the zeros of the function.

41) $f(x) = x^4 - 5x^3 + 6x^2 + 54x - 108$ 41) ______

A) {-3, 2, 3 + 4i, 3 - 4i} B) {-3, 2, 3 + 3i, 3 - 3i}

C) {-2, 3 + 3i, 3 - 3i} D) {3, -2, $3 + \sqrt{3}$, $3 - \sqrt{3}$}

Based on the solutions shown, two of $\{-3, -2, 2, 3\}$ are roots. Let go directly to them. (Descartes Rule of Signs does not give us any additional information since there are lots of sign changes.)

Let's try synthetic division with 2 as a root (or "zero").

$$\begin{array}{r|rrrrr} 2 & 1 & -5 & 6 & 54 & -108 \\ & & 2 & -6 & 0 & 108 \\ \hline & 1 & -3 & 0 & 54 & \boxed{0} \end{array}$$

So, 2 is a root of the original polynomial.

Looking at the solutions, the only solution with 2 as a root is Answer **B**. Let's select it and move on.

Answer B

Find the vertical asymptotes, if any, of the graph of the rational function.

42) $g(x) = \frac{x}{x^2 - 9}$ 42) ______

A) $x = 3, x = -3$ B) $x = 3, x = -3, x = 0$
C) $x = 3$ D) no vertical asymptote

Vertical asymptotes exist at all roots of the denominator that remain after the function is factored and simplified.

$$g(x) = \frac{x}{x^2 - 9} = \frac{x}{(x-3)(x+3)}$$

There is no more factoring or simplifying to be done. So, we look at the roots of the denominator for where the vertical asymptotes are.

$(x-3)(x+3) = 0$, so, vertical asymptotes exist at $\boldsymbol{x = 3,\ x = -3}$. **Answer A**

Find the slant asymptote, if any, of the graph of the rational function.

43) $f(x) = \frac{x^2 + 9x - 9}{x - 3}$ 43) ______

A) $y = x$ B) $y = x + 9$
C) $y = x + 12$ D) no slant asymptote

A slant asymptote exists whenever the numerator is one degree higher than the denominator. That is the case with this function, so a slant asymptote exists. To obtain the slant asymptote, divide the numerator by the denominator and disregard any fractional component of the answer.

Use synthetic division to obtain the slant asymptote. Recall that the divisor term is the root of the denominator of the function, i.e., 3. Then,

$$\begin{array}{c|ccc} 3 & 1 & 9 & -9 \\ & & 3 & 36 \\ \hline & 1 & 12 & \boxed{27} \end{array}$$

The result of the division, excluding the fractional component provides the slant asymptote:

$\boldsymbol{y = x + 12}$ **Answer B**

Write the equation in its equivalent exponential form.

44) $\log_b 8 = 3$ 44) ______

A) $8^b = 3$ B) $3^b = 8$ C) $b^3 = 8$ D) $8^3 = b$

In the log expression, $\log_b 8 = 3$, **first** is "b", **last** is "3" and **middle** is "8." We put these in an exponential expression, from left to right, to get: $\boldsymbol{b^3 = 8}$. **Answer C**

Write the equation in its equivalent logarithmic form.

45) $7^3 = x$ 45) _______

A) $\log_x 7 = 3$ B) $\log_7 x = 3$ C) $\log_7 3 = x$ D) $\log_3 x = 7$

In the exponential expression, $7^3 = x$, **first** is "7", **last** is "x" and **middle** is "3." We put these in a logarithmic expression, from left to right, to get: $\mathbf{\log_7 x = 3}$. **Answer B**

Solve the problem.

46) The long jump record, in feet, at a particular school can be modeled by $f(x) = 20.2 + 2.5 \ln (x + 1)$ where x is the number of years since records began to be kept at the school. What is the record for the long jump 7 years after record started being kept? Round your answer to the nearest tenth. 46) _______

A) 25.1 feet B) 25.4 feet C) 24.7 feet D) 22.7 feet

$f(x) = 20.2 + 2.5\ln(x + 1)$

$f(7) = 20.2 + 2.5\ln(7 + 1) = \mathbf{25.4\ feet}$. **Answer B**

Use properties of logarithms to expand the logarithmic expression as much as possible. Where possible, evaluate logarithmic expressions without using a calculator.

47) $\ln\left(\frac{e^3}{2}\right)$ 47) _______

A) $\ln e^3 - \ln 2$ B) $3 - \ln 2$ C) $3 + \ln 2$ D) $\ln e^3 + \ln 2$

Recall for this problem that the natural log function and e to a power are inverses. Then,

$$\ln\left(\frac{e^3}{2}\right) = \ln e^3 - \ln 2 = \mathbf{3 - \ln 2}$$ **Answer B**

Use properties of logarithms to condense the logarithmic expression. Write the expression as a single logarithm whose coefficient is 1. Where possible, evaluate logarithmic expressions.

48) $\log_6 (x - 2) - \log_6 (x + 6)$ 48) _______

A) $\log_6 -8$ B) $\log_6 (x^2 + 4x - 12)$

C) $\log_6\left(\frac{x - 2}{x + 6}\right)$ D) $\log_6\left(\frac{x - 2}{x - 6}\right)$

When adding logs, multiply the arguments.

When subtracting logs, divide the arguments.

$$\log_6(x - 2) - \log_6(x + 6) = \mathbf{\log_6\left(\frac{x - 2}{x + 6}\right)}$$ **Answer C**

Solve the logarithmic equation. Be sure to reject any value that is not in the domain of the original logarithmic expressions. Give the exact answer.

49) $5 + 9 \ln x = 12$ 49) ______

A) $\{e^{7/9}\}$ B) $\left\{\ln \frac{7}{9}\right\}$ C) $\left\{\frac{7}{9 \ln 1}\right\}$ D) $\left\{\frac{e^7}{9}\right\}$

Starting equation:	$5 + 9\ln x = 12$
Subtract 5:	$9 \ln x = 7$
Divide by 9:	$\ln x = \frac{7}{9}$
Take e to the power of both sides:	$e^{\ln x} = e^{7/9}$
Simplify:	$x = e^{7/9}$ **Answer A**

50) $\log_6 (x^2 - 5x) = 1$ 50) ______

A) {6} B) {1} C) {6, −1} D) {−6, 1}

Starting equation:	$\log_6(x^2 - 5x) = 1$
Take 6 to the power of both sides:	$6^{\log_6(x^2-5x)} = 6^1$
Simplify:	$x^2 - 5x = 6$
Subtract 6:	$x^2 - 5x - 6 = 0$
Factor:	$(x-6)(x+1) = 0$
Determine solutions for x:	$x = \{6, -1\}$
Test the solutions of x:	$x = 6$: $\log_6(6^2 - 5 \cdot 6) = 1$ ✓
	$x = -1$: $\log_6\left((-1)^2 - 5 \cdot (-1)\right) = 1$ ✓

Note: To test the solutions you derive, use the original equation or a simplified form of the original equation.

Final solution: $\mathbf{x = \{-1, 6\}}$ **Answer C**

Note: It is unusual that both solutions are valid, but in this case they are. You must always check each solution to see if it is valid!

Solve.

51) The population of a particular country was 21 million in 1980; in 1993, it was 31 million. The exponential growth function $A = 21e^{kt}$ describes the population of this country t years after 1980. Use the fact that 13 years after 1980 the population increased by 10 million to find k to three decimal places. 51) ______

A) 0.030 B) 0.177 C) 0.040 D) 0.498

There is only one step to this problem:

1) Find the value of k based on the population change from 1980 to 1993.

What are the variables?

The formula for exponential decay is: $A = A_0 e^{kt}$, where:

- A is the population at time t.
- A_0 is the starting population.
- k is the annual rate of growth.
- t is the number of years.

Step 1: Determine the value of k

We are given: $A = 31$, $A_0 = 21$, $t = 13$

Starting equation: $A = A_0 e^{kt}$

Substitute in values: $31 = 21e^{k \cdot 13}$

Divide by 21: $\frac{31}{21} = e^{13k}$

Take natural logs: $\ln\left(\frac{31}{21}\right) = 13k$

Divide by 13: $\boldsymbol{k} = \frac{\ln\left(\frac{31}{21}\right)}{13} = \mathbf{0.0299588}$

Round to 3 decimal places: $\boldsymbol{k} = \mathbf{0.030}$ **Answer A**

Solve the exponential equation. Express the solution set in terms of natural logarithms.

52) $e^{x + 5} = 8$ 52) ______

A) $\{\ln 8 - 5\}$ B) $\{\ln 13\}$ C) $\{e^8 + 5\}$ D) $\{e^{40}\}$

$e^{x+5} = 8 \quad \Rightarrow \quad x + 5 = \ln 8 \quad \Rightarrow \quad \boldsymbol{x = -5 + \ln 8}$ **Answer A**

Use common logarithms or natural logarithms and a calculator to evaluate to four decimal places

53) $\log_9 16$ 53) ______

A) 1.2619 B) 0.2499 C) 2.1584 D) 0.7925

We need the change of base formula for this one: $\log_b a = \frac{\ln a}{\ln b} = \frac{\log a}{\log b}$

$\log_9 16 = \frac{\ln 16}{\ln 9} = \mathbf{1.2619}$ **Answer A**

Evaluate the expression without using a calculator.

54) $e^{\ln 2x^5}$ 54) ______

A) $2x^5$ B) $\ln 2x^5$ C) 5 D) e^{2x^5}

Recall for this problem that e to a power and the natural log function are inverses. Then,

$e^{\ln(2x^5)} = \mathbf{2x^5}$ **Answer A**